Interplay between chiral and axial symmetries in a SU(2) Nambu–Jona-Lasinio Model with the Polyakov loop

M. C. Ruivo,[*] M. Santos,[†] P. Costa,[‡] and C. A. de Sousa[§]

Departamento de Física, Universidade de Coimbra,

P-3004-516 Coimbra, Portugal, EU

(Dated: December 30, 2011)

Abstract

We consider a two flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model where the Lagrangian includes an interaction term that explicitly breaks the $U_A(1)$ anomaly. At finite temperature, the restoration of chiral and axial symmetries, signaled by the behavior of several observables, is investigated. We compare the effects of two regularizations at finite temperature, one of them, that allows high momentum quarks states, leading to the full recovery of chiral symmetry. From the analysis of the behavior of the topological susceptibility and of the mesonic masses of the axial partners, it is found in the SU(2) model that, unlike the SU(3) results, the recovery of the axial symmetry is not a consequence of the full recovery of the chiral symmetry. Thus, one needs to use an additional idea, by means of a temperature dependence of the anomaly coefficient, that simulates instanton suppression effects.

PACS numbers: 11.30.Rd, 11.55.Fv, 14.40.Aq

[*] Corresponding authorM. C. Ruivo; maria@teor.fis.uc.pt

[†] mario@teor.fis.uc.pt

[‡] pcosta@teor.fis.uc.pt

[§] celia@teor.fis.uc.pt

I. INTRODUCTION

Spontaneous breaking and restoration of chiral symmetry is one of the most fundamental aspects inferred from quantum chromodynamic theory (QCD) and is manifested through the equation of state of hot and dense hadronic matter. The Nambu–Jona-Lasinio (NJL) model is one of the most studied effective models of QCD at finite temperature and has been applied successfully to reproduce the low-energy phenomena, although this model does not support some features of QCD, such as quark confinement.

With the intention of broadening the applicability of effective QCD models, in particular, allowing for the implementation of additional features of the QCD phenomenology, the Polyakov Loop is considered within the NJL model, that so is endowed with a mechanism that allows the analysis of the expected confinement/deconfinement phase transition in the hadron-quark system. This extended model, the so called Polyakov–Nambu–Jona-Lasinio (PNJL) model, first implemented in Ref. [1], provides a simple framework which considers the chiral and the confinement order parameters. In fact, the NJL model describes the interactions between constituent quarks, hence providing the correct chiral properties. The static gluonic degrees of freedom, introduced in the NJL Lagrangian through an effective gluon potential in terms of the Polyakov loop, take into account features of deconfinement [1–4]. A good approach to reproduce lattice results is given by the coupling of the quarks to the Polyakov loop. This leads to a reduction weight of the quark degrees of freedom as the critical temperature is approached from above (interpreted as a manifestation of confinement).

Besides the chiral symmetry, another important symmetry, explicitly broken in the QCD Lagrangian, is the axial $U_A(1)$ symmetry [5], which might also be restored at extreme temperatures. In fact, there are indications from the lattice calculations that, at high temperatures, effects arising from the $U_A(1)$ breaking are strongly suppressed. This suggests an effective restoration of the $U_A(1)$ symmetry. An important issue that the present paper intends to analyze is the possible restoration of axial and chiral symmetries, which observables may give indications regarding this restoration, and whether there is an interplay between both restorations or not. By this we will use the PNJL model in the SU(2) sector but with the inclusion of a term on the Lagrangian that explicitly breaks the $U_A(1)$ symmetry. It is worth noting that the effects of the anomaly have been intensively studied in the NJL model in

SU(3) with the 't Hooft term [6–10], and more recently, the possibility of axial symmetry restoration and its effects on diverse observables [6, 9–11]. The model in SU(2), due to its simplicity, allows to isolate some aspects of the problem, reason why its study can lead to a relevant contribution for the understanding of physics associated to the breaking and restoration of the $U_A(1)$ symmetry. Interest in studying this issue in two flavor models is manifested in recent works [12].

The same model has been used in Ref. [13] which particularly looks into the nature of the superfluid phase. Here we will focus on the topological susceptibility, the chiral and axial partners and the comparison between the results of SU(2) and SU(3) models.

We will investigate the behavior of the following observables with temperature: quark condensates, topological susceptibility and π, σ, a_0 and η mesons. Two approaches will be followed in order to understand the mechanism of restoration of axial symmetry: we will consider different degrees of $U_A(1)$ symmetry breaking in the vacuum, by using different values for the strength of the anomaly that are kept constant with varying temperature; alternatively, we fix the degree of symmetry breaking in the vacuum and allow the coupling strength to decrease with temperature. We will also consider the effects upon the observables behavior of using two types of regularization:

- *Regularization I*: as the cutoff Λ is only necessary to regularize some integrals in the vacuum and it is not necessary at finite T, it is considered infinite as soon as thermal effects are considered ($\Lambda \to \infty$). This has the effect of allowing for high quark moments;

- *Regularization II*: the cutoff is taken always with a finite value, at zero or finite T ($\Lambda = const.$), and all the momenta couple with the same strength up to a cutoff momentum $\Lambda \sim 1$ GeV.

In Sec. II we will introduce the formalism for a two flavor NJL type model with two different interacting parts that allow to disentangle chiral and axial symmetries. The gap equations and the mesonic propagators will also be discussed, as well as the breaking of the axial symmetry and the topological susceptibility. The Polyakov loop extension of the model will be explained as well. The different types of regularization will be presented in the context of the model extension for finite temperature. We redirect the details of the calculations to the appendices included in a previous paper [10]. In Sec. III we present and discuss our results for the phase transition and also for the temperature behavior of the meson masses and the topological susceptibility, aiming at discussing restoration of

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symmetries. We conclude in Sec. IV with a brief summary.

II. FORMALISM

The aim of this section is to present the mathematical formalism of the NJL model and its extension to the PNJL model. In the present work, a two flavor, three color, quark system will be studied, and it will be explicitly introduced a term on the Lagrangian that acts as a source of anomaly. The anomaly of the SU(2) model is already present in the original NJL model [14, 15] through a Fierz transformation and has been considered in other works as well (see Ref. [16] and Refs. there in), the strength of the anomaly coupling being the same as the chiral coupling. In the present work we have an extra term that ensures an independent mechanism of axial symmetry breaking.

A. A SU(2) Nambu–Jona-Lasino type model

The Lagrangian that will be used here is a SU(2) version of the NJL model taking into account an additional term that, although being a chiral invariant, explicitly breaks the axial symmetry.

We start with a NJL type model with two flavors defined by the following Lagrangian:

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mathcal{L}_1 + \mathcal{L}_2, \tag{1}$$

with two different interacting parts

$$\mathcal{L}_1 = g_1 \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 + (\bar{q}\vec{\tau}q)^2 + (\bar{q}i\gamma_5 q)^2 \right], \tag{2}$$

$$\mathcal{L}_2 = g_2 \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 - (\bar{q}\vec{\tau}q)^2 - (\bar{q}i\gamma_5 q)^2 \right]. \tag{3}$$

The quark fields $q = (u, d)$ are defined in Dirac and color fields, respectively with two flavors, $N_f = 2$ and three colors, $N_c = 3$, the coupling constants g_1 and g_2 have dimension $energy^{-2}$, and $\hat{m} = \text{diag}(m_u, m_d)$ is the current quark mass matrix.

The Lagrangian is chiral invariant in the limit where the current quark masses vanish. Both terms $\mathcal{L}_1$ and $\mathcal{L}_2$ are invariant upon $\text{SU}(2)_L \otimes \text{SU}(2)_R \otimes \text{U}(1)$ type transformations, but

the $\mathcal{L}_2$ component makes the Lagrangian non-covariant upon $U_A(1)$ transformations. The $\mathcal{L}_2$ term, that may be represented in the form of a determinant:

$$\mathcal{L}_2 = 2g_2\Big[\det\big[\bar{q}(1+\gamma_5)q\big] + \det\big[\bar{q}(1-\gamma_5)q\big]\Big], \tag{4}$$

can be identified as an interaction induced by instantons, according to 't Hooft, and it explicitly breaks the axial symmetry even in the chiral limit.

In this model, while the vector current is conserved, the axial isovector current is only conserved in the chiral limit and, as a consequence of the $\mathcal{L}_2$ term in the Lagrangian, the isoscalar axial current is not conserved even in the chiral limit. In fact:

$$\partial_\mu j_5^\mu = 2m(\bar{q}\ i\gamma_5 q) + 8g_2\Big[(\bar{q}\vec{\tau}q)(\bar{q}i\gamma_5\vec{\tau}q) - (\bar{q}q)(\bar{q}i\gamma_5 q)\Big]. \tag{5}$$

The last equation is the equivalent of the QCD 4-divergence of j_5^μ:

$$\partial_\mu j_5^\mu = 2m(\bar{q}\ i\gamma_5 q) + 2N_F Q(x), \tag{6}$$

which allows to identify the topological charge $Q(x)$ in our model as:

$$Q(x) = 2g_2\Big[(\bar{q}\vec{\tau}q)(\bar{q}i\gamma_5\vec{\tau}q) - (\bar{q}q)(\bar{q}i\gamma_5 q)\Big], \tag{7}$$

or, equivalently, as

$$Q(x) = 2g_2\Big[\det\big[(\bar{q}(1-\gamma_5)q - \det\big[(\bar{q}(1+\gamma_5)\big]. \tag{8}$$

This equation shows the important role played by the coupling constant g_2 in our analysis bearing in mind that the physical effects of the $U_A(1)$ anomaly are only manifested with non-zero topological charge.

Having identified the topological charge, the topological susceptibility may be calculated [11], as it will be shown in Sec. II.C.

The four fields in the original Lagrangian can be rearranged in the form:

$$\mathcal{L}_1 + \mathcal{L}_2 = \frac{g_s}{2}\Big[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\Big] + \frac{g_a}{2}\Big[(\bar{q}\vec{\tau}q)^2 + (\bar{q}i\gamma_5 q)^2\Big], \tag{9}$$

where $g_s = 2(g_1 + g_2)$ and $g_a = 2(g_1 - g_2)$.

From the last expression it is easy to understand the reason why the Lagrangian supports four mesonic channels: $\mathcal{O}_\sigma = \mathbb{1}$, $\mathcal{O}_\pi = i\gamma_5\vec{\tau}$, $\mathcal{O}_\eta = i\gamma_5$ and $\mathcal{O}_{\vec{a_0}} = \mathbb{1}\vec{\tau}$. After integrating the

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generating functional over quark fields and using $\sigma, \vec{\pi}, \eta, \vec{a_0}$ as auxiliary fields, the following effective action is obtained:

$$I_{eff} = -i\,\mathrm{Tr}\,\ln\left(-i\slashed{\partial} - m + \sigma + i\gamma_5\vec{\tau}\vec{\pi} + i\gamma_5\eta + \vec{\tau}\vec{a_0}\right) - \frac{\sigma^2 + \vec{\pi}^2}{2g_s} - \frac{\eta^2 + \vec{a_0}^2}{2g_a}. \quad (10)$$

A standard calculation leads straightforwardly to the gap equation and to the meson propagators. The interaction term of the Lagrangian with g_s coupling is responsible for the calculation of the propagators of the chiral partners (π, σ), while the term associated to g_a allows for the chiral partners (η, a_0). Since we consider equal quark masses, flavor mixing effects induced by the axial symmetry breaking are not visible.

The following gap equations are obtained:

$$M_i = m_i - 2g_s\,\langle\bar{q}q\rangle_i, \quad (11)$$

where one identifies $i = u, d$ and M_i as the constituent quark mass. The quark condensates are determined by

$$\langle\bar{q}q\rangle_i = -i\mathrm{Tr}\frac{1}{\hat{p} - M_i} = -i\mathrm{Tr}S_i(p), \quad (12)$$

being $S_i(p) = (\slashed{p} - M_i + i\varepsilon)^{-1}$ the propagator of quarks.

The mass spectra of the mesons is obtained by the analysis of the pole structure of the meson propagator, given by

$$1 - 2g_{s,a}\,\Pi_{\mathcal{M}}(q^2 = M_{\mathcal{M}}^2) = 0. \quad (13)$$

where

$$\Pi_{\mathcal{M}}(q2) = i\int\frac{d^4p}{(2\pi)4}\mathrm{Tr}\left[\mathcal{O}_{\mathcal{M}}\,S(p+q)\,\mathcal{O}_{\mathcal{M}}\,S(p)\right] \quad (14)$$

is the polarization operator for the quark-antiquark system regarding the channel with quantum numbers $\{\mathcal{M}\}$ in the mesonic sector. As mentioned above, g_s is related to π and σ mesons and g_a to η and a_0 mesons.

B.   Extension to the PNJL model

Now we include the Polyakov loop and its effective potential to the NJL type model described above. The Lagrangian of this SU(2)⊗SU(2) quark model with explicit chiral

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a_0	a_1	a_2	b_3
3.51	-2.47	15.2	-1.75

TABLE I. Parameters for the effective potential in the pure gauge sector.

symmetry breaking where the quarks couple to a (spatially constant) temporal background gauge field (represented in term of Polyakov loops) is given by [3, 17]:

$$\mathcal{L}_{PNJL} = \bar{q}\,(\,i\,\gamma^\mu\,D_\mu\,-\,\hat{m}\,)\,q + \frac{g_s}{2}\,[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + \frac{g_a}{2}\,[(\bar{q}\vec{\tau}q)^2 + (\bar{q}i\gamma_5 q)^2]$$
$$-\,\mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right). \tag{15}$$

The quarks are coupled to the gauge sector *via* the covariant derivative $D^\mu = \partial^\mu - iA^\mu$. The strong coupling constant g_{Strong} has been absorbed in the definition of A^μ: $A^\mu(x) = g_{Strong}\mathcal{A}^\mu_a(x)\frac{\lambda_a}{2}$ where $\mathcal{A}^\mu_a$ is the $SU_c(3)$ gauge field and λ_a are the Gell–Mann matrices. Besides in the Polyakov gauge and at finite temperature $A^\mu = \delta^\mu_0 A^0 = -i\delta^\mu_4 A^4$.

The Polyakov loop Φ (the order parameter of $\mathbb{Z}_3$ symmetric/broken phase transition in pure gauge) is the trace of the Polyakov line defined by: $\Phi = \frac{1}{N_c}\langle\langle \mathcal{P}\exp i\int_0^\beta d\tau\, A_4\left(\vec{x}, \tau\right)\rangle\rangle_\beta$.

The pure gauge sector is described by an effective potential $\mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right)$ chosen to reproduce at the mean-field level the results obtained in lattice calculations:

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}; T\right)}{T^4} = -\frac{a\,(T)}{2}\bar{\Phi}\Phi + b(T)\ln[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2], \tag{16}$$

where

$$a\,(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad \text{and} \quad b\,(T) = b_3\left(\frac{T_0}{T}\right)^3. \tag{17}$$

The effective potential exhibits the feature of a phase transition from color confinement ($T < T_0$, the minimum of the effective potential being at $\Phi = 0$) to color deconfinement ($T > T_0$, the minima of the effective potential occurring at $\Phi \neq 0$).

The parameters of the effective potential $\mathcal{U}$ are given in Table I. These parameters have been fixed in order to reproduce the lattice data for the expectation value of the Polyakov loop and QCD thermodynamics in the pure gauge sector [18, 19].

The parameter T_0 is the critical temperature for the deconfinement phase transition within a pure gauge approach: it was fixed to 270 MeV, according to lattice findings. This choice ensures an almost exact coincidence between chiral crossover and deconfinement at zero chemical potential, as observed in lattice calculations.

The PNJL grand canonical potential density in the $SU_f(2)$ sector can be written as [3, 20]:

$$\Omega(\Phi, \bar{\Phi}, M; T, \mu) = \mathcal{U}\left(\Phi, \bar{\Phi}, T\right) + 2g_s N_f \langle \bar{q}_i q_i \rangle^2 - 2N_c N_f \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)\,3} E_p$$

$$- 2N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left(z_\Phi^+(E_i) + z_\Phi^-(E_i)\right), \tag{18}$$

where E_i is the quasi-particle energy for the quark i: $E_i = \sqrt{\mathbf{p}^2 + M_i^2}$, and z_Φ^+ and z_Φ^- are the partition function densities.

The explicit expression of z_Φ^+ and z_Φ^- are given by:

$$z_\Phi^+(E_i) \equiv \mathrm{Tr}_c \ln \left[1 + L^\dagger e^{-\beta(E_i+\mu)}\right] = \ln \left\{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_i+\mu)}\right) e^{-\beta(E_i+\mu)} + e^{-3\beta(E_i+\mu)}\right\},$$
$$\tag{19}$$

$$z_\Phi^-(E_i) \equiv \mathrm{Tr}_c \ln \left[1 + L e^{-\beta(E_i-\mu)}\right] = \ln \left\{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta(E_i-\mu)}\right) e^{-\beta(E_i-\mu)} + e^{-3\beta(E_i-\mu)}\right\}.$$
$$\tag{20}$$

A word is in order to describe the role of the Polyakov loop in the present model. Almost all physical consequences of the coupling of quarks to the background gauge field stem from the fact that in the expression of z_Φ, Φ or $\bar{\Phi}$ appear only as a factor of the one- or two-quarks (or antiquarks) Boltzmann factor, for example $e^{-\beta(E_i-\mu)}$ and $e^{-2\beta(E_i-\mu)}$. Hence when $\Phi, \bar{\Phi} \to 0$ (signaling what we designate as the "confined phase") only $e^{-3\beta(E_i-\mu)}$ remains in the expression of the grand canonical potential, leading to a thermal bath with a small quark density. At the contrary $\Phi, \bar{\Phi} \to 1$ (in the "deconfined phase") gives a thermal bath with all 1-, 2- and 3-particle contributions and a significant quark density.

This formalism, presented here for completeness in the grand canonical approach, will be employed in the present work with $\mu = 0$. This condition implies $\Phi = \bar{\Phi} = 0$.

C. The topological susceptibility

The topological susceptibility, χ, is an essential parameter for the study of the breaking and restoration of the $U_A(1)$ symmetry.

The topological susceptibility is defined as:

$$\chi = \int \mathrm{d}^4 x \, \langle 0|TQ(x)Q(0)|0\rangle_c, \tag{21}$$

where c means connected diagrams and T the time order operator. Using the definition of

Eq. (7) it can be seen that the operator $Q(x)Q(0)$ in this model reads:

$$
\begin{aligned}
Q(x)Q(0) &= 4g_2^2 \Big[[\bar{q}(x)q(x)][\bar{q}(x)\gamma_5 q(x)] - [\bar{q}(x)\vec{\tau}q(x)][\bar{q}(x)\gamma_5\vec{\tau}q(x)] \Big] \\
&\qquad \Big[[\bar{q}(0)q(0)][\bar{q}(0)\gamma_5 q(0)] - [\bar{q}(0)\vec{\tau}q(0)][\bar{q}(0)\gamma_5\vec{\tau}q(0)] \Big] \\
&= 4(G\alpha)^2 \Big[[\bar{q}(x)q(x)][\bar{q}(x)\gamma_5 q(x)][\bar{q}(0)q(0)][\bar{q}(0)\gamma_5 q(0)] \\
&\quad - [\bar{q}(x)q(x)][\bar{q}(x)\gamma_5 q(x)][\bar{q}(0)\vec{\tau}q(0)][\bar{q}(0)\gamma_5\vec{\tau}q(0)] \\
&\quad - [\bar{q}(x)\vec{\tau}q(x)][\bar{q}(x)\gamma_5\vec{\tau}q(x)][\bar{q}(0)q(0)][\bar{q}(0)\gamma_5 q(0)] \\
&\quad + [\bar{q}(x)\vec{\tau}q(x)][\bar{q}(x)\gamma_5\vec{\tau}q(x)][\bar{q}(0)\vec{\tau}q(0)][\bar{q}(0)\gamma_5\vec{\tau}q(0)] \Big].
\end{aligned} \tag{22}
$$

Taking into account only the connected diagrams of order $1/N_c$ (see Fig. 1) and following a similar approach to [11], we arrive at the following expression for the topological susceptibility:

$$
\chi = 4N_f \, g_2^2 \, \langle \bar{q}q \rangle^2 \, \frac{4I_1}{1 - 8g_a I_1}. \tag{23}
$$

As already mentioned, one of the aims of this paper is to perform a comparison between the behavior with temperature of the topological susceptibility in the framework of the SU(2) and SU(3) PNJL models with explicit axial symmetry breaking. As we will see, some meaningful differences will appear, so it is useful, in order to understand these differences, to compare both expressions. For the sake of simplicity we will write the SU(3) expression for the case of equal quark masses $m_u = m_d = m_s$:

$$
\chi = 4N_f \, (g_D \langle \bar{q}q \rangle)^2 \, \langle \bar{q}q \rangle^2 \, \frac{4I_1}{1 - 8P_{00}I_1}, \tag{24}
$$

where $P_{00} = (g_S - 2g_D \langle \bar{q}q \rangle)$ (for details concerning the SU(3) model see the Appendix A and Ref. [21]). A comparison shows that the expressions for quantities with effects of the anomaly in SU(2) depend on a constant, g_2, while the corresponding expressions in SU(3) depend on a constant multiplied by a quark condensate, $g_D \langle \bar{q}q \rangle$. The same replacement holds for the gap equations and for the propagators of the chiral partners (η, a_0).

We will return to this point again when discussing the behavior of several observables with temperature where it will be shown that some effects are different in SU(2) and SU(3) due to the fact that in the last model the "effective" constant $g_{D_{eff}} = g_D \langle \bar{q}q \rangle$ already depends on temperature through the quark condensate, while its corresponding in SU(2), g_2, remains constant, unless we explicitly allow a variation with temperature.
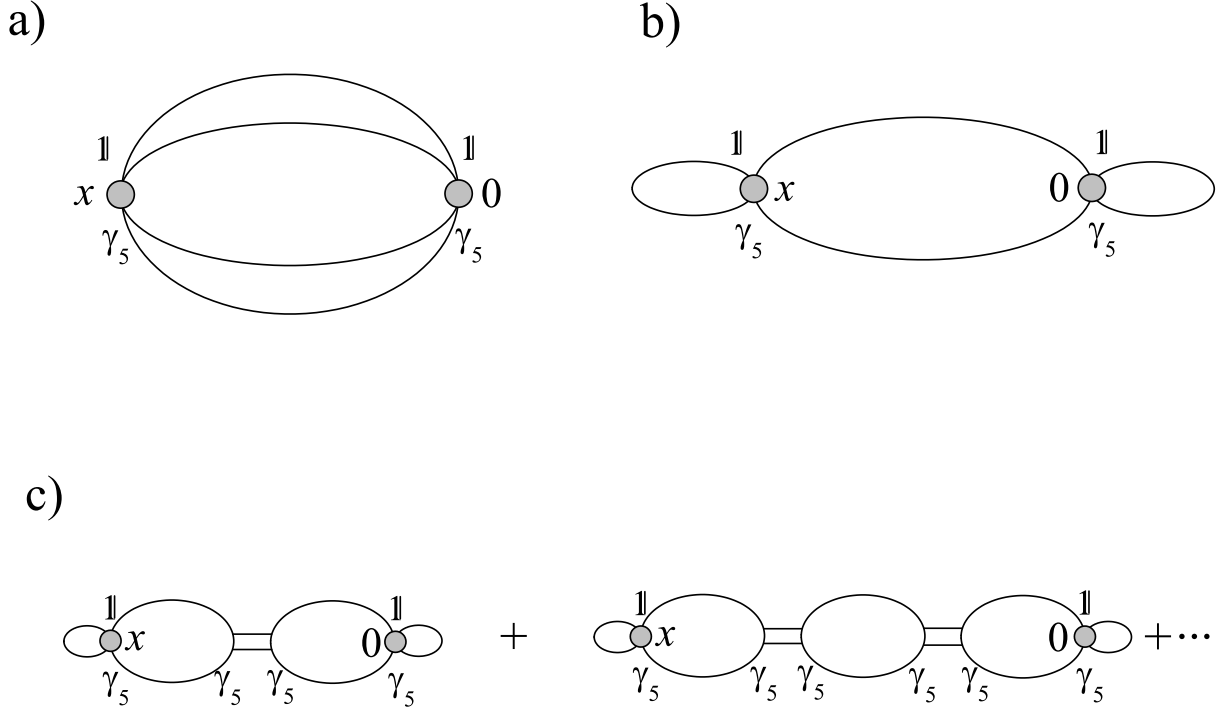
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FIG. 1. Connected diagrams for the loop ((a) and (b)) and for the ring contributions (c). Only (b) and (c) are of order $1/N_c$.

III. RESULTS

A. Vacuum properties and parameters fixing

The present PNJL model has four parameters in the NJL sector: m, Λ, g_1 and g_2. We choose to adjust the parameters in vacuum by fitting to well known experimental data or lattice values: the mass of the pion, its decay constant, the quark condensate and the topological susceptibility, χ, that are shown in Table II. The masses of the σ, η and a_0 mesons come as outputs.

Our main purpose is to study the behavior of several observables with temperature, in particular those that might signal the chiral and deconfinement phase transitions, and the effective restoration of chiral and axial symmetries. For this we will use two different

	f_π [MeV]	$\langle \bar{q}q \rangle^{1/3}$ [MeV]	m_π [MeV]	m_σ [MeV]	m_η [MeV]	m_{a_0} [MeV]	$\chi^{1/4}$ [MeV]
Model	93	-300	140.2	803.7	704.5	919.8	180.8
Exp. /Latt.	92.4	-270	135.0	400-1200	547.3	984.7	180

TABLE II. Numerical values for the calculated observables: f_π, $\langle \bar{q}q \rangle$, the meson masses and the topological susceptibility, obtained with $\Lambda = 590$ MeV, $G\Lambda^2 = 2.435$, $\alpha = 0.2$ and $m = 6$ MeV.

approaches, that will be presented now in the vacuum, and its consequences for the behavior with temperature will be explored in the next subsection. The two scenario to be considered are:

Scenario A - We will keep g_1 and g_2 as independent parameters. At finite temperature we may allow g_2 to be temperature dependent but g_1 is kept constant. This is equivalent to the usual treatment of the SU(3) model;

Scenario B - We redefine the coupling constants such as the set (g_1, g_2) will be replaced by (G, α) in the following parametrization:

$$g_1 = G\,(1 - \alpha), \qquad g_2 = G\,\alpha, \tag{25}$$

with $\alpha \in \{0, 1\}$ [13, 16]. In this picture g_1 and g_2 are not independent, but $g_s = 2(g_1 + g_2) = 2G$ will be kept always constant; on the contrary, $g_a = 2G(1 - 2\alpha)$ varies with α.

Consequently, when studying the effect of the degree of axial symmetry breaking, by choosing different values for α, only the topological susceptibility and the masses of η, a_0 will be affected, since they depend explicitly on α; the π and σ channels will not be affected.

Let us now discuss *Scenario B* for the vacuum state considering special values for the parameter α:

(1) $\alpha = 0$: the coupling constant $g_2 = 0$ and the axial symmetry is unbroken.

(2) $\alpha = 1$: the coupling constant $g_1 = 0$ and we have a pure instanton interaction, where $U_A(1)$ is broken maximally.

(3) $\alpha = 0.5$: in this case $g_1 = g_2 = G$, the standard NJL model [16] is recovered, and only the σ and π channels, are reproduced.

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α	$\chi^{1/4}(180)$ [MeV]	$\eta(547)$ [MeV]	$a_0(984)$ [MeV]
0	0	140.2	803.7
0.10	150.7	510.2	856.7
0.112	155.5	547.0	863.7
0.16	170.6	641.0	893.2
0.20	180.8	704.5	919.8
0.25	191.5	766.1	956.3
0.30	200.7	799.3	996.4
0.40	216.0	986.9	1094.3
0.50	228.6	–	–

TABLE III. Numerical values for the topological susceptibility, and masses of the mesons η and a_0 for different values of the anomaly parameter, α.

In order to see the effects of the degree of $U_A(1)$ symmetry breaking in the vacuum, we allow α to take values between 0 and 0.5. The results are summarized in Table III. As it can be seen, for $\alpha = 0$ (no anomaly) the η is degenerated in mass with the pion, the a_0 with σ, and the topological susceptibility is zero, as expected. A small breaking of the axial symmetry ($\alpha = 0.1$) is enough to break the degeneracy, raising the masses of η (that comes close to almost its experimental value) and a_0, and to get a meaningful non-vanishing value of $\chi^{1/4}$. Larger values of α yield larger values of $\chi^{1/4}$ and of the meson masses. When $0.16 \leq \alpha \leq 0.25$ the value of χ is within the range of values coming from lattice results [22, 23].

1. Behavior with temperature

Let us now discuss our results at finite temperature within the SU(2) PNJL model with anomaly. We aim at analyzing signals of possible restoration of chiral and axial symmetries and also to compare these results with those obtained within the corresponding SU(3) model [21, 24]. First we analyze the characteristic temperatures for the chiral/deconfinement phase transition. An important question to address is whether $U_A(1)$ is effectively restored at finite
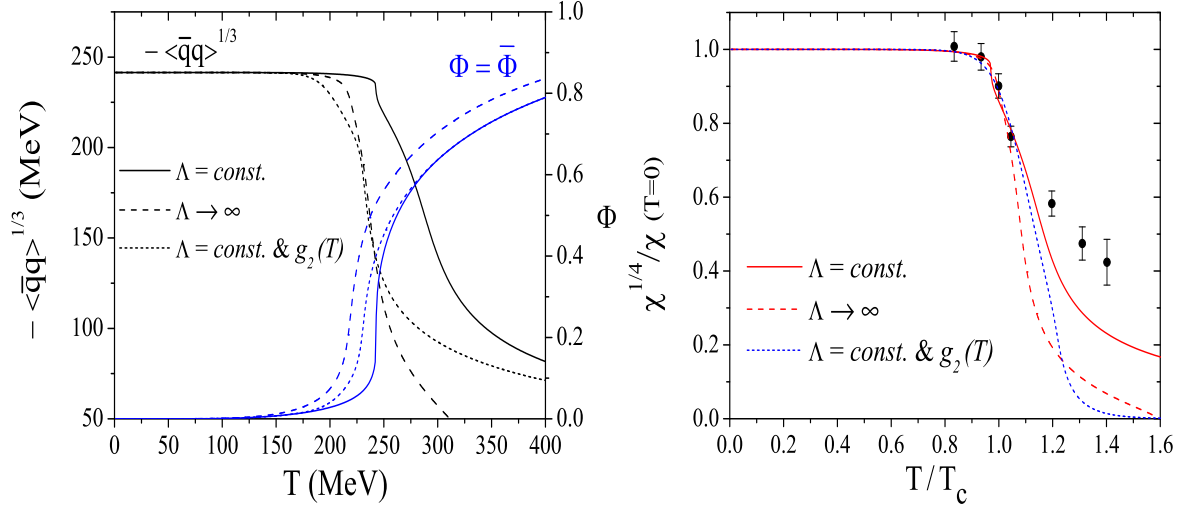
FIG. 2. Comparison of the behavior with temperature of $-\langle\bar{q}q\rangle^{1/3}$ and the Polyakov field Φ (left panel) and of the topological susceptibility (right panel) with g_2 constant and two regularizations (*I*: $\Lambda \to \infty$, *II*: $\Lambda = const.$) and with g_2 a decreasing function of temperature and *regularization II* ($\alpha = 0.2$). The lattice points for χ were taken from Ref. [22].

temperature and its possible relation with the restoration of chiral symmetry. To quantify this effect it is mandatory to analyze the behavior of the topological susceptibility, and of the mesonic excitations with temperature. Another important question in this concern is to discuss if χ should be considered as an order parameter for the axial symmetry. A challenging question is whether the restoration of axial symmetry arises as consequence of the restoration of chiral symmetry, or if there is an independent mechanism for the suppression of instanton effects. To simulate the last effect, the anomaly coupling g_2 may be assumed as a decreasing function of temperature.

We start by discussing the behavior with temperature of observables that may signal restoration of chiral symmetry, deconfinement and restoration of axial symmetry in different situations. For this purpose, let us first analyze in Fig. 2 (left panel) the behavior of the quark condensate $-\langle\bar{q}q\rangle^{1/3}$, of the Polyakov field Φ (left panel) and of the topological susceptibility (right panel) with g_2 constant and two *regularizations* (*I*: $\Lambda \to \infty$, *II*: $\Lambda = const.$). From now on we will focus our discussion for the case $\alpha = 0.2$ (the analysis is qualitatively similar for others values of α.)

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The results for the quark condensate and for the Polyakov loop are similar to those within the SU(3) model. As it was shown in [6, 25], the critical temperature for the chiral and deconfinement transitions, identified with the derivatives of the quark condensate and of the Polyakov field, respectively, become closer using *regularization I*. As shown in [3, 24] this regularization, together with the Polyakov loop, is very convenient to describe the thermodynamic properties of the system leading to a good agreement with lattice results. It has the disadvantage, for high temperatures, of leading to a sharp decrease of the quark condensate that, when using *regularization I*, vanishes and, unless an additional condition is implemented, becomes negative.

In order to discuss the influence of an independent mechanism of suppression of instantons with temperature, we consider a third situation: the anomaly coupling is assumed to decrease with T, $g_2(T) = g_2(0)e^{-(T-300)/20}$, and *regularization II* is used. Since *Scenario A* is considered, the variation of g_2 with T implies also a variation of $g_s = 2(g_1 + g_2)$, and therefore the "mechanism of instanton suppression" influences the restoration of chiral symmetry and deconfinement. In fact, we can see from Fig. 2 that both $-\langle \bar{q}q \rangle^{1/3}$ and Φ are affected by the decrease of g_2. By examining the values of Table IV we see that, concerning the critical temperatures, both this case and *regularization I* with g_2 constant lead to a greater proximity of the deconfinement and chiral phase transitions as compared with *regularization II* and g_2 constant. We emphasize that the ansatz $g_2(T)$ leads even to equal values for T_c^χ and T_c^Φ.

Observing the right panel of Fig. 2, we see that, as already shown in the SU(3) model [21], PNJL calculation for the topological susceptibility nicely reproduces the first lattice points, a feature that is not verified in the NJL model [21]. The effects of the introduction of high quark moments through *regularization I* is only relevant at high temperatures, allowing a decrease of χ, that eventually becomes zero. The vanishing of χ is also achieved with a decreasing with temperature of the strength of the anomaly $(g_2(T))$. So, after analyzing both figures, it seems that two independent mechanisms, the allowance of hight momentum quark states and the instanton suppression, produce some similar effects: proximity between chiral and deconfinement phase transitions, and sharp decrease and vanishing of the topological susceptibility. The question is whether the mentioned behavior of the topological susceptibility is enough to signal restoration of the axial symmetry.

With rising temperature we expect the chiral $SU(2)_V \otimes SU(2)_A \simeq SU(2)_L \otimes SU(2)_R$ sym-

	regularization I $(\Lambda \to \infty)$	regularization II $(\Lambda = const.)$	regularization II with $g_2(T)$
T_c^χ [MeV]	237	287	231
T_c^Φ [MeV]	219	243	231
T_c [MeV]	228	265	231
T_{eff}^χ [MeV]	$\simeq 260$	$\simeq 350$	$\simeq 245$
T_{eff}^A [MeV]	—	—	$\simeq 285$

TABLE IV. Critical temperatures for chiral and deconfinement phase transitions (T_c^χ and T_c^Φ; T_c is the average between both) and for the effective restoration of chiral and axial symmetries (T_{eff}^χ and T_{eff}^A), obtained for $\alpha = 0.2$. The values in the two first rows were obtained with finite and infinite cutoff, respectively, and g_2 constant; in the third row g_2 is a decreasing function of temperature and the cutoff is kept constant.

metry to be restored, and the chiral partners (π, σ) and (η, a_0) become degenerate in mass, the effective restoration of chiral symmetry being then achieved.

Concerning the axial symmetry, the vanishing of the mass splitting of the axial chiral partners (π, a_0) and (σ, η) is a necessary condition for the restoration of this symmetry. In order to discuss this problem, we are going to analyze the behavior with temperature of the topological susceptibility and meson masses by exploring two situations: (i) using the two regularizations already presented, but without allowing the anomaly coupling g_2 to decrease with temperature, (ii) considering several degrees of violation of axial symmetry *ab initio* by using three different values for the anomalous parameter α. Once again we will consider a fixed $\alpha = 0.2$ for $T = 0$.

The temperature dependence of the mesonic observables is displayed in Fig. 3. Concerning the chiral partners (σ, π), we observe that in the vicinity of 260 (350) MeV for *regularization I* (*regularization II*) both masses become identical, a sign for the effective restoration of chiral symmetry. At a temperature slightly lower there occurs the convergence of the chiral partners (η, a_0). The interesting result is that the restoration of the axial symmetry is not achieved: the masses of the two partners (π, σ) and (η, a_0), although getting close at high temperatures, do not converge. Concerning the variation of α, the lower values of this parameter favor the restoration of axial symmetry, as expected, since
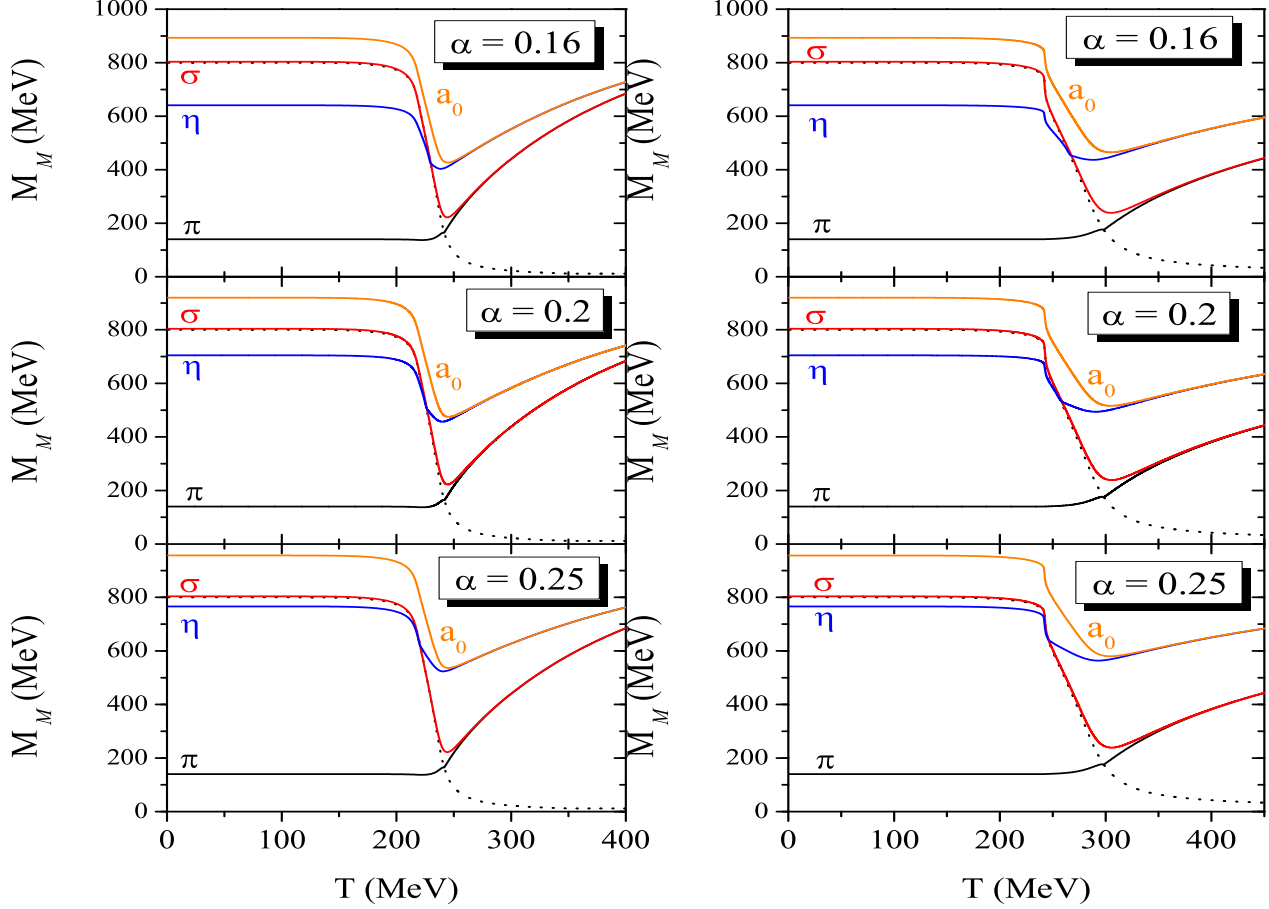
FIG. 3. Masses of the meson chiral partners (π, σ) and (η, a_0) with varying temperature for three different values of the anomaly parameter α with two regularizations at finite T: *regularization I* (left panel) and *regularization II* (right panel). The dotted lines represent the $q\bar{q}$ threshold $2M_u$.

the masses of the axial chiral mesons were already closer in the vacuum. Even when high momentum quarks are allowed (left panel) the masses of partners, although closer, do not converge (see Fig. 3, right panel), in spite of the vanishing of the topological susceptibility in this case (see Fig. 4).

This result is different from the one obtained in the SU(3) versions of NJL and PNJL models [21], where one could obtain a simultaneous vanishing of the splitting of the axial partners and of the topological susceptibility when *regularization I* was used. In fact, the inclusion of strange quarks ($N_f = 3$) leads to non-linear (mixing) effects since the $U_A(1)$ anomaly term is trilinear for three flavors and thus would generate additional mechanisms of restoration of axial symmetry, even if a constant anomaly strength is taken into account. In
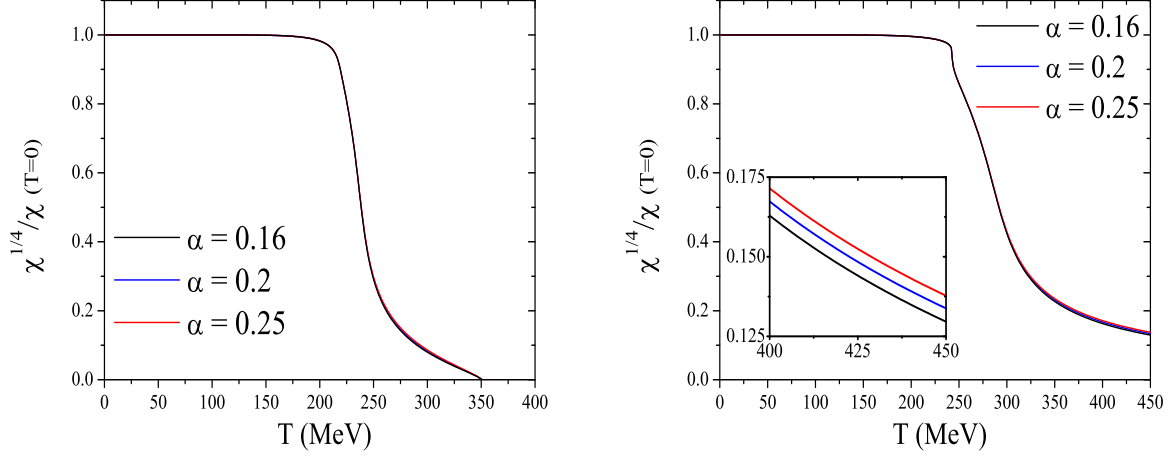
FIG. 4. Normalized topological susceptibility as function of temperature for several values of the anomaly parameter α with *regularizations I* (left panel) and *II* (right panel).

order to understand the present results, let us remember the comparison of the expressions for χ within the SU(3) and SU(2) models as discussed in Sec. II C.

A meaningful difference between both models is that in SU(3) we can define an effective anomaly strength, $g_{D_{eff}} = g_D \langle \bar{q}q \rangle$, that, as the temperature varies, acquires a temperature dependence through the variation with temperature of the quark condensate. Therefore it is possible to obtain restoration of chiral axial symmetry without additional assumptions, the infinite cutoff at finite T leads to the vanishing of the quark condensate at high temperatures and, as a consequence of the effective anomaly strength and of the quantities that depends on it: the topological susceptibility and the mass splitting between the axial partners. In this case restoration of axial symmetry is a mere consequence of the full restoration of chiral symmetry. It is not necessary to implement an anzats related to explicit instanton suppression. As we will show, the vanishing of the mass splitting between axial partners in SU(2) demands such a mechanism.

The results plotted in Figs. 2–4 already give an indication that a weaker axial symmetry breaking (in this case obtained by hand, by giving low values to α) favors the restoration of axial symmetry and reduces the mass splitting of the axial partners (see Fig. 3). In fact, the restoration of this symmetry can only be modeled in a phenomenological way by making g_2 temperature dependent. Therefore we will calculate the meson mass spectrum
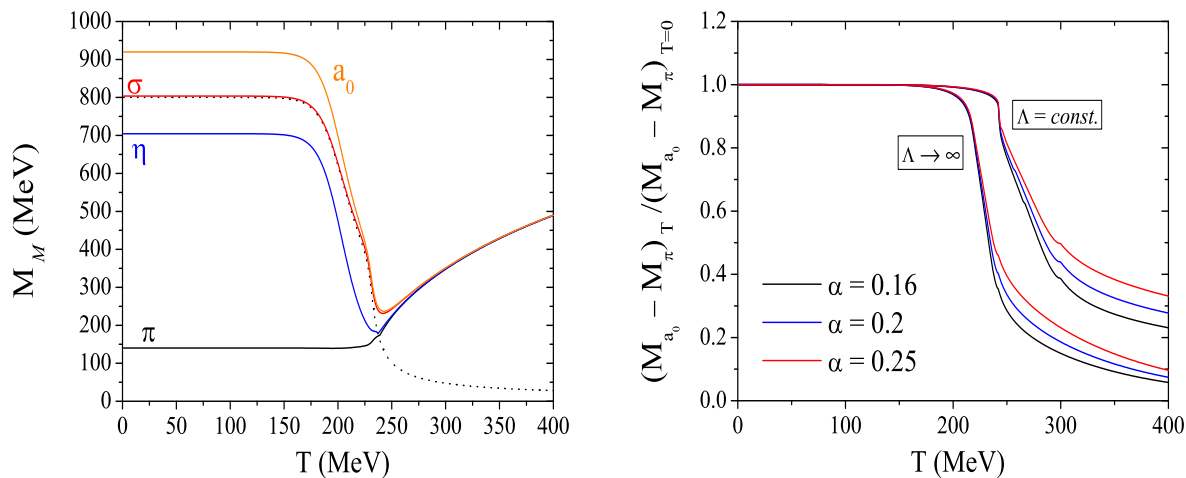
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FIG. 5. Left panel: masses of the meson chiral partners (π, σ) and (η, a_0) with varying temperature for $g_2(T)$ as a decreasing function of temperature, using *regularization II* ($\Lambda = const.$) at finite T. The dotted line represents the $q\bar{q}$ threshold $2M_u$. Right panel: normalized mass splitting of axial partners as function of temperature for different values of α.

with g_1 fixed and g_2 a decreasing exponential of the temperature, $g_2(T) = g_2(0)e^{-(T-300)/20}$. The result is plotted in Fig. 5 where we can see that, although the effective restoration of chiral symmetry (vanishing of the mass splitting between σ and π, and between η and a_0) occurs first, at $T = 245$ MeV, the degeneracy of the π, η, a_0 and σ occurs at $T = 285$ MeV. Therefore, although the analysis of the effects of high momentum quark states and of the instanton suppression have similarities in what concerns the characteristic temperatures for the phase transitions and the high temperature behavior of the topological susceptibility, the evidence that they are indeed mechanisms with a different physical meaning appears when we look for a possible degeneracy of the axial chiral partners.

IV. CONCLUSIONS

The present study is dedicated to the analysis of the behavior of various observables that signal the restoration of the chiral and axial symmetries in regards to temperature, within a SU(2) Polyakov–Nambu–Jona-Lasinio model with a 't Hooft interaction term.

Two types of regularization of finite temperature were considered; the first regularization

consists in using the cutoff only on divergent integrals (I) and the other consists in always using a finite cutoff (II). The effect of the anomalous coefficient dependent on temperature, $g_2(T)$, was also analyzed. The restoration of chiral and axial symmetries with temperature was studied, considering the effects in the symmetry restoration process of the type of regularization applied. A comparative study between the results here obtained and the ones calculated within the PNJL model in SU(3) [6] was also performed. Similarly to SU(3), it was verified in the present work that the critical temperatures obtained with *regularization I* are closer to lattice results than with *regularization II*; however, the effective restoration of the chiral and axial symmetries does not occur simultaneously.

We hence conclude that when the coupling constants are considered constant, the restoration of the axial symmetry, in the SU(2) sector, does not occur as a natural consequence of the complete restoration of the chiral symmetry, being this fact a fundamental difference between the SU(3) and the SU(2) sectors, where it is not sufficient to apply the *regularization I* for all the observables, related with the $U_A(1)$ symmetry, to vanish. It is therefore necessary to use an additional mechanism for this restoration, such as a dependence on the temperature of the coupling coefficient that is related to the anomaly. Once g_2 is the coupling constant of the term that breaks the axial symmetry and simulates the instanton effect, this result seems to indicate that, in SU(2), the instantons are not completely suppressed by the restoration of the chiral symmetry.

A relevant contribution that the present study offers for the understanding of physics associated with the breaking and restoration of the $U_A(1)$ symmetry refers to the fact that the analysis of the topological susceptibility, χ, is the "necessary condition", but not the "sufficient condition", for the study and comprehension of the restoration of the axial symmetry.

Appendix A

Some meaningful differences between the results within the SU(3) and the SU(2) models appear, so it is useful, in order to understand these differences, to present the basic ingredients of the SU(3) model. The SU(3) NJL model with the 't Hooft determinant has the following Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - \hat{m}\right)q + \frac{g_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}\,i\gamma_5\lambda^a\,q)^2\right]$$
$$+ g_D\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right], \tag{A1}$$

which can be rewritten as:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - \hat{m}\right)q + \frac{1}{2}\left\{(\bar{q}\lambda^a q)S_{ab}(\bar{q}\lambda^b q) + (\bar{q}\,i\gamma_5\lambda^a\,q)P_{ab}(\bar{q}\,i\gamma_5\lambda^b\,q)\right\}, \tag{A2}$$

where the following projectors have been introduced:

$$S_{ab} = g_S\delta_{ab} + g_D D_{abc}\langle\bar{q}\lambda^c q\rangle, \tag{A3}$$

$$P_{ab} = g_S\delta_{ab} - g_D D_{abc}\langle\bar{q}\lambda^c q\rangle. \tag{A4}$$

$\langle\bar{q}\lambda^c q\rangle$ being the vacuum expectation values. The constants D_{abc} coincide with the SU(3) structure constants d_{abc} for a, b, c = (1, 2, . . . , 8) and $D_{0bc} = -\frac{1}{\sqrt{6}}\delta_{bc}$ and $D_{000} = \sqrt{\frac{2}{3}}$.

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